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Reflection of electromagnetic pulses from a moving inhomogeneous plasma*

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The problem of reflection of a Gaussian envelope amplitude modulated (AM) pulse from an inhomogeneous plasma, moving uniformly in a direction parallel to the plane of incidence, has been analysed. The electron density in the plasma is assumed to vary linearly such that the moving plasma is described by a linearly decreasing specific permittivity profile. Some numerical results have been presented which explicitly show the dependence of amplitude reflection coefficient and the reflected pulse on the velocity of the inhomogeneous plasma and the angle of incidence.

1 INTRODUCTION

In this communication, we have investigated the problem of reflection of a Gaussian envelope AM electromagnetic pulse from an inhomogeneous plasma, moving in a direction parallel to the interface and to the plane of incidence (i.e. moving in x-direction, x–z being the plane of incidence). The plasma is assumed to be inhomogeneous in the direction of propagation of the pulse (i.e., z-direction). The problem has been analyzed by the method based on Maxwell's equations and the constitutive relations for a moving medium known as Maxwell-Minkowski approach; in a similar manner as done by Tanaka & Hazama (1972) for a dispersionless medium.

2 FORMULATION AND RESULTS

Assuming a linearly increasing electron density profile in the plasma, the specific permittivity of the plasma moving with uniform velocity $v = c\beta$, in x-direction is given by

$$\epsilon_p(Z) = 1 - \omega_p{}'^2(E)/\omega'_p{}^2, \qquad Z > 0 \qquad \ldots (1)$$

$$= 1, \qquad Z < 0 \qquad \ldots (2)$$

with

$$\omega'_p{}^2(E) = \omega^2(E) = 4\pi e^2 \alpha Z/m$$
$$\omega' = \gamma\omega(1-\beta\sin\theta)$$
$$\gamma^{-2} = 1-\beta^2$$

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where $-e$ and m are charge and mass of the electron; α, is the linear inhomogeneity parameter of the plasma; c, is the velocity of light in vacuum and θ, is the angle of incidence of an electromagnetic wave incident on the free-space moving plasma interface.

Now the amplitude reflection coefficient of a plane E-wave propagating in free space and incident on a medium characterized by eq. (1) at an angle θ; is well known (Yeh & Liu 1972), and is

$$R = \frac{\cos\theta\, A_i(\xi_0) - i(a/k)^{1/3} A'_i(\xi_0)}{\cos\theta\, A_i(\xi_0) + i(a/k)^{1/3} A'_i(\xi_0)} \quad .. \quad (3)$$

with

$$\xi_0 = -(k/a)^{2/3}$$

$$k/a = \frac{k_0\gamma(1-\beta\sin\theta)\left[1-\left(\frac{\beta-\sin\theta}{1-\beta\sin\theta}\right)^2\right]^{3/2}}{[4\pi c^2\alpha/m\gamma^2\omega^2(1-\beta\sin\theta)^2]}$$

$$k_0 = \omega/c.$$

where $A_i(\xi_0)$ and $A'_i(\xi_0)$ are the Airy function and its derivative with respect to z. It is worthwhile to note that under the condition $k/a \gg 1$ (i.e., WKB approximation) which holds good for many cases of interest, the above expression for the amplitude reflection coefficient simplifies to

$$R \simeq \exp\left[-i\left(\frac{4\omega^3 B}{3} - \pi/2\right)\right] \quad \ldots \quad (4)$$

where

$$B = \frac{m}{4\pi c^2 c\alpha}\left[\gamma(1-\beta\sin\theta)\right]^3 \cdot \left[1-\left(\frac{\beta-\sin\theta}{1-\beta\sin\theta}\right)^2\right]^{3/2}$$

Let the incident source pulse be the Gaussian envelope AM pulse given by

$$E_i(t) = \exp(-\sigma^2 t_i^2)\cos(\omega_0 t_i) \quad \ldots \quad (5)$$

whose frequency spectrum is

$$E_i(\omega) = (2\pi)^{-1} \int_{\infty}^{\infty} E_i(t) e^{-i\omega t_i} dt$$

$$= \frac{\sqrt{\pi}}{2\sigma}\left[e^{-(\omega-\omega_0)^2/4\sigma^2} + e^{-(\omega-\omega_0)^2/4\sigma^2}\right] \quad .. \quad (6)$$

where $t_i = t-(x \sin\theta - z\cos\theta)/c$, ω_0 is the carrier frequency and σ is the standard deviation related to the width of the pulse.

If the transfer function of the moving inhomogeneous plasma is given by

$$|R(\omega)| \exp[-i\Phi(\omega)]$$

then the reflected pulse is

$$E_\gamma(t_\gamma) = (2\pi)^{-1} \int_{-\infty}^{\infty} |R(\omega)| E_i(\omega) e^{i[\Phi(\omega)-\omega t\gamma]} \, d\omega \qquad \ldots (7)$$

where

$$t_\gamma = t-(x \sin\theta + z\cos\theta)/c$$

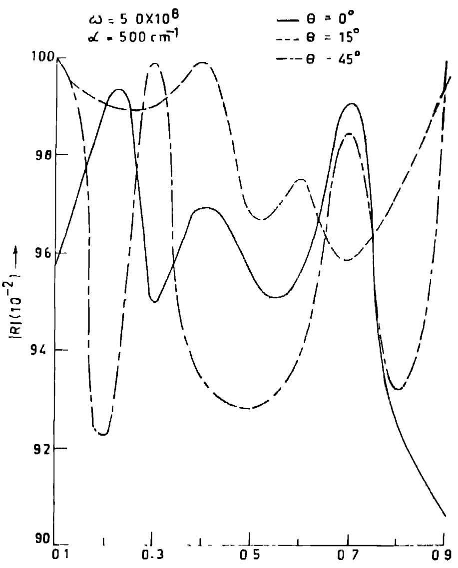

Fig. 1. Amplitude reflection coefficient as a function of velocity for various angles of incidence.

Using eqs (4), (6) and (7) and using the concept of a wave packet i.e., we assume that the energy of the source pulse is concentrated in a relatively narrow band of frequencies around the carrier frequency (Ginzburg 1970), the reflected pulse takes the form

$$E_y(t_y) = |R(\omega_0)| \frac{e^{-[\sigma(t_y-\Phi')]^2/\left[1+\left\{\frac{\Phi''}{2}(2\sigma)^2\right\}^2\right]}}{\left[1+\left\{\frac{\Phi''}{2}(2\sigma)^2\right\}^2\right]^{1/4}}$$

$$\times\cos\left[\omega_0 t_y-\Phi(\omega_0)-\tfrac{1}{2}\tan^{-1}\left\{\frac{\Phi''}{2}(2\sigma)^2\right\}+\frac{[\sigma(t_y-\Phi')]^2}{\left[1+\left\{\frac{\Phi''}{2}(2\sigma)^2\right\}^2\right]^{1/4}}\frac{\Phi''}{2}(2\sigma)^2\pm\frac{\pi}{2}\right] \qquad \dots \text{(8)}$$

where

$$|R(\omega_0)| = 1$$

$$\Phi(\omega_0) = \frac{4B\omega_0^3}{3} - \pi/2$$

$$\Phi'(\omega_0) = \left.\frac{d\Phi}{d\omega}\right|_{\omega=\omega_0} = 4B\omega_0^2.$$

and

$$\Phi''(\omega_0) = \left.\frac{d^2\Phi}{d\omega^2}\right|_{\omega=\omega_0} = 8B\omega_0.$$

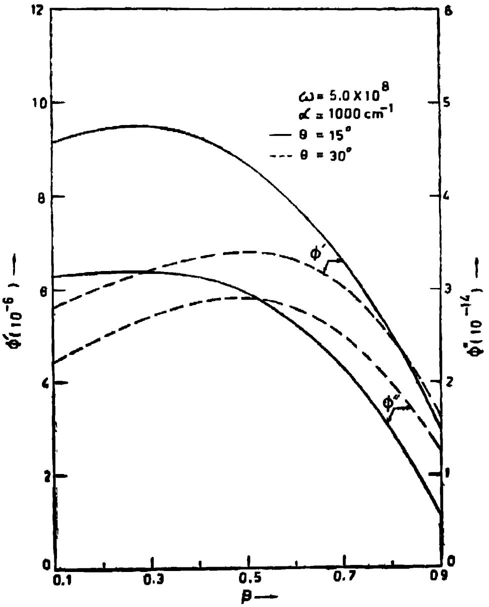

Fig. 2. Pulse delay, $\Phi'(\omega_0)$, and pulse distortion parameter, $\phi''(\omega_0)$, as a function of velocity for various angles of incidence.

A close inspection of eq (8) shows that the pulse is shifted on time axis after reflection and the whole pulse is delayed by pulse delay time, $\Phi'(\omega_0)$. Let us define a para- meter l_0 as

$$l_0 = \Phi''(\omega_0)(2\sigma)^2/2$$

Which represents the distortion of the pulse after suffering reflection. In case $l_0 \gg 1$, the carrier waveform is no longer Gaussian, but in case $l_0 \ll 1$, the envelope of the reflected pulse remains Gaussian.

Figure 1 shows the variation of amplitude reflection coefficient, $|R|$, with velocity parameter β of the moving plasma and θ, the angle of incidence, $|R|$, shows an oscillatory behaviour (due to oscillating nature of the Airy function) whose maximum value approaches unity as the numerator is complex—conjugate of the denominator in eq (3). Figures 2 and 3 show the variation of group delay time, $\Phi'(\omega_0)$, and the distortion parameter $\Phi''(\omega_0)$ (since $l_0 \alpha \Phi''(\omega_0)$ for a given value of σ) with θ and α. Both $\Phi'(\omega_0)$ and $\Phi''(\omega_0)$ decrease, (a) with increase in the angle of incidence, (b) with increase in the value of inhomogeneity parameter.

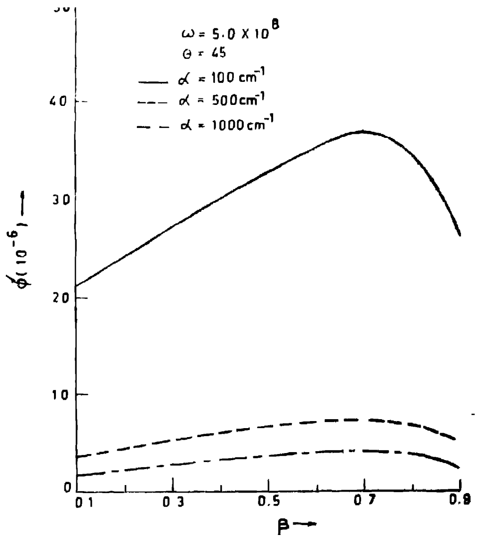

Fig. 3 Pulse delay, $\Phi'(\omega_0)$, as a function of velocity for various values of inhomogeneity parameter, α.

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